THE PEDAGOGICAL CONTENT KNOWLEDGE OF LATIN-AMERICAN CHEMISTRY PROFESSORS ON THE MAGNITUDE «AMOUNT OF SUBSTANCE» AND ITS UNIT «MOLE»

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Abstract

This paper seeks to address the following question: Can college professors' Pedagogical Content Knowledge (PCK) of the topic «amount of substance» be properly documented by Loughran *et al.*'s methodology, and therefore be classified and discussed with the help of Mortimer's conceptual profile? The PCK of the topic «amount of substance» of four university professors in General Chemistry from Argentina and Mexico are hereby documented, following both the Content Representation (CoRe) and the Professional and Pedagogical experience Repertoires (PaP-eRs) methodologies. In order to evaluate professor's structure of knowledge we managed to apply the results obtained in the CoRe with Mortimer's conceptual profile model, in three steps. The first consisted of selecting, by agreement among those professors interviewed, the main central ideas involved in the teaching of «amount of substance», a fundamental magnitude of the International System of Units (SI). Secondly, five conceptual profile zones were defined, following the guidelines proposed by Mortimer: Perceptive/intuitive, empiricist, formalist, rationalist and formal rationalist. Finally, these zones were used as criteria to classify each one of the sentences provided by each professor in the CoRe frame of questions, from which individual conceptual profile graphs were constructed. Such graphs include both the epistemological and ontological commitments of individual teacher, and offer an enlightening way towards the classification of professors' knowledge base from which their teaching characteristics can be analyzed and discussed. A couple of PaP-eRs are included in the appendices of this work.

Introduction

Pedagogical Content Knowledge (PCK)

Shulman (1986, p. 9) introduced PCK as a specific category of knowledge, one "which goes beyond knowledge of subject matter *per se* to the dimension of subject matter knowledge *for teaching*". While the concept of PCK has been debated in the literature (Gess-Newsome and Lederman, 1999), it is generally agreed that the development of PCK is embedded in classroom practice (Van Driel, Verloop and De Vos, 1998). Ideally, teachers should be familiar with students' alternative conceptions and learning difficulties, and be able to organize, arrange, deliver and assess subject matter. Skilful teachers (Barnett & Hodson, 2001) transform subject matter into forms more accessible to students, adapting it to the specific learning context and thereby developing their own PCK. Since Shulman (1986) coined this term, several methods designed to document teacher PCK have been suggested and then classified into three categories (Baxter and Lederman, 1999): (1) Convergent and inferential techniques (*e.g.*, multiple-choice tests); (2) concept mapping, card sorts, and pictorial representations; and (3) multi-method evaluations (Van Driel and De Jong, 2001).

One of the most successful ways to document PCK is Loughran, Mulhall and Berry's (2004) CoRes and PaP-eRs proposal. We have used this methodology in several previous researches and have found it is a very interesting and appropriate method of portraying and documenting PCK. After choosing the most important teaching ideas of the specific content, the professors answered a frame of questions (called Content Representation, CoRe), which allowed us to explore their PCK. The evaluation of answers intends to find out the professor's teaching objectives; his/her knowledge of student's alternative conceptions; those problems that commonly arise in students when learning; the most effective sequence in which to arrange the topic elements and any important approaches to the framing of each idea; the appropriate use of analogies, demonstrations and examples; and any insightful ways of testing for understanding, among other pedagogic factors. The most relevant results are presented as a set of arrays or matrixes, one for each professor, so the researcher may compare and relate their constituent elements. On the other hand, PaP-eRs (Professional and Pedagogical experience Repertoires) are narrative essays reflecting individual teachers experience and drawn from classroom observations or teacher interviews.

In a later article, Shulman (1987) included PCK in what he called "the knowledge base for teaching". This knowledge base was built of seven categories, three of which were content-related (*i.e.*, subject matter knowledge, PCK, and curriculum knowledge). The remaining four categories referred to general pedagogy, learners and their characteristics, educational contexts, and educational purposes.

PCK is concerned with the teaching of specific topics, and may differ considerably from subject matter knowledge of these same topics. However, researchers have pointed out that it is not always possible to make a sharp and clear distinction between PCK and subject matter knowledge (Tobin, Tippins and Gallard, 1994), insisting on the pedagogic nature of subject matter knowledge (McEwan and Bull, 1991; Segall, 2004).

Conceptual Profile Model

Mortimer (1995) has proposed a way to analyze conceptual evolution in the classroom: The conceptual profile model. It suggests that there may be different ways of thinking in different domains, each one represented by a conceptual profile zone ranging from common-sense to scientific ideas. Learning science does not entail the replacement of intuitive ideas by scientific ones, but rather the slow and progressive change from a

given conceptual profile to a new one with a more complex knowledge level. We have used this model to characterize a pattern of teaching emphasis manifested in a given moment by one specific professor that fully exhibited his/her epistemological and ontological commitments.

Content

The subject matter of this work is «amount of substance» and its unit, the «mole». It was selected for this research due to its importance as a magnitude in chemistry, and also due to the difficulty in teaching and learning it. Dierks (1981) reported that 300 journal papers on this topic had appeared up to that date.

This concept, introduced internationally around 1961, has been defined in the following way:

> "The amount of substance is proportional to the number of specified elementary entities of that substance; the proportionality factor is the same for all substances and is the reciprocal of the Avogadro constant." (Mills *et al.*, 1993: P. 46)

Recently, the IUPAC adopted the synonymous «chemical amount», proposed by Gorin (1994), due to the fact that critics have pointed out that sometimes a given «amount of substance» does not refer to any substance at all —as in the case of a mole of electrons, radicals or ions. Nevertheless we will refer to it as «amount of substance», because it is a common practice nowadays.

The lack of knowledge we as teachers have concerning the socio-historical context of this concept and of the evolution of its meaning after the adoption of the atomic-molecular theory by modern chemistry may in part be blamed for the difficulties in its teaching. That is why several key papers on the topic point to the lack of a historical context as one of the main causes of the problems related to the teaching of the concept of the «mole» (Dierks, 1981; Strömdahl *et al.*, 1994; Furió *et al.*, 2000). The term «mole» emerged in an equivalentist theoretical frame; due to the meaning that Ostwald gave it as a chemical combination weight —although in a strict sense he always used the term «amount of substance» in its meaning of mass. The modern atomistic theoretical frame considers «amount of substance» as the result of macroscopically counting unimaginable elementary units.

Almost all professors resort at least once to the unit «mole» during the General Chemistry course. Since Johnstone *et al.* (1971) illuminated the teaching of the «mole» as a source of learning difficulties for chemistry students; a lot of research has been done on this concept and on the troubles associated with its teaching and learning (Novick and Menis, 1976; Kolb, 1978; Dierks, 1981; Nelson, 1991; Strömdahl *et al.*, 1994; Furió *et al.*, 2000). The results of the present work will be analyzed with regard to and compared with this quoted studies.

The teacher needs to distinguish clearly between «amount of substance» (n), mass (m), volume (V) and number of elementary entities (N). Furió *et al.* (2000) found that the introduction of the «mole» concept made in most chemistry textbooks wrongly attributes to it the meanings of chemical mass (50.6% of the texts) and/or of number of elementary entities (21.8%).

Objectives of This Study

In this study we try to find out if it is possible to obtain the college professors' way of thinking related with the topic «amount of substance» by using a methodology that combines Loughran *et al.*'s CoRe and Mortimer conceptual profile model. In order to do so, we combined these two research methods; the first one —Loughran's *et al.* method— helped us to document the PCK of chemistry professors related to «amount of

substance» and the second one —Mortimer's method— assisted us to classify, construct and represent the commitments of each professor by means of their conceptual profiles. One of our tools to search for the professors' way of thinking is related to chemistry history and «mole» concept development. Some professors have always taught this concept as a mass instead of a macroscopic measurement of certain set of elementary entities. That means they have taught it inside the equivalentist paradigm even if they believe on atoms and molecules existence and this is a contradictory way of thinking. Our perception is that professors use both paradigms without knowing certainly which one is better for the learning-teaching process. They have just used which is more convenient to theirs believes and biases.

Methodology

General Characteristics of the Four Professors Surveyed

The main characteristics of the four professors surveyed were:
They were two females and two males, the four of them working full time in either a Mexican or an Argentinean university. To preserve the anonymity of the subjects, all educators are designated from here onward as "she" regardless of their gender. Professor 1 has 15 years of teaching experience. She has a PhD degree and did a postdoctoral stay at a renowned foreign university. Professors 2 and 3 both earned B.S. degrees and individually have more than 30 years of teaching experience. Professor 4 has a PhD degree and almost 30 years of teaching experience.

Adaptation of Loughran et al.'s (2004) Documentation of PCK

PCK remains a seductive theoretical construct, but not an easily identifiable aspect of practice; consequently, there are lacks of readily available concrete examples of PCK in the literature. Although PCK exists in the teachers' mind, it is a difficult process both to articulate and document for numerous reasons (Baxter and Lederman, 1999; Loughran, Mulhall and Berry, 2004):

a) A teacher's PCK may not be evident within the bounds of a few lessons; as it is a complex notion, an extended period of time may be needed to unfold it.
b) Observations can provide only limited insight into a teacher's PCK, because it is partly an internal construct. Science teachers do not use a language that resembles the construct of PCK, and much of their knowledge of practice is tacit.
c) Teachers commonly share activities, teaching procedures, and clever insights into teaching and learning with implicit purposes in practice, but rarely express the reasons behind them.

The method developed by Loughran, Mulhall and Berry (2004) to uncover, document, and portray science teachers' PCK comprises two tools: Content Representation (CoRe) and Pedagogical and Professional experience Repertoires (PaP-eRs). Both are methods designed to capture PCK and portray this knowledge to others.
In relation with the CoRe, the first task for the teachers is to consider what they believe to be the main ideas in teaching the particular content issue (the so-called central ideas). Afterwards, by means of a set of eight framing questions for each central idea, they are required to describe how they help their students to understand and assess these ideas.
It is convenient to clarify the meaning given to the term «central ideas». Mulhall, Berry and Loughran (2003) state:

> "«Big ideas» is a term often used in science to describe an idea that has had a profound impact on the ways scientists understand and conceptualize the world.

> Our use of the term «central ideas» is not synonymous with this: we mean the science ideas that a teacher sees as being at the heart of understanding the topic for the particular class under consideration".

As Loughran *et al.* do we manage to have a unique set of «central ideas» by doing a consensus among three of the professors surveyed. After receiving and evaluating the professors' sets of individual «central ideas», a common set of ideas was selected by the researchers, which was then re-examined with the three professors until an agreement was reached. After repeating two times the aforementioned process, we finally arrived to the following consensus «central ideas»:

- Fundamental magnitudes of the International System of Units. «Amount of substance».
- Relative atomic mass.
- «Mole», the unit of «amount of substance».
- Molar mass.
- Avogadro's hypothesis and molar volume.
- Number of elementary entities and Avogadro's constant.

Afterwards, by means of a set of eight framing questions, they were required to describe how they help their students to understand and assess these ideas. For the purposes of this research, modifications were done to the frame utilized by Loughran *et al*. (2004), specifically in the CoRe section: it was considered that more items (such as historical, epistemological, philosophical, and STS-related) exist in regard to the content studied, that the original questionnaire did not take into account and was deemed worthwhile to include. In our research we adapted the original questions (see Table 1 for the set of questions used in this research to document the CoRe section).

Table 1. Questions made to interviewed professors, which constitute the frame of the Content Representation in this research.

1. Why is it important for students to learn this idea and what do you intend in teaching it?
2. From an STS and historical point of view, why is it important for students to learn this?
3. What else do you know about the history, philosophy and epistemology of this idea?
4. Difficulties/limitations connected with learning this idea
5. Difficulties/limitations connected with teaching this idea
6. Knowledge about students' thinking, which influences your teaching of this idea
7. Teaching procedures for engaging students with this idea (analogies, metaphors, examples, demonstrations, reformulations, etc.)
8. Specific ways for ascertaining students' understanding or confusion about this idea

Once the answers from each professor to these eight questions were compiled, an array or matrix of cells was constructed (with the central ideas in the first row and the questions in the first column. The i^{th} row and j^{th} column contain the answers to question i in regard with what central idea j reveals).

Conceptual Profile

The sort of information that can be obtained from the CoRe is the professor's thinking while lecturing, which in turn is related to her ways of teaching and the procedures used for specific contents. The authors believe that there is enough information in the CoRes to find out the epistemological and ontological commitments of individual teachers, as

represented by the conceptual profile proposed by Mortimer (1995), derived in turn from Bachelard's (1940) epistemological profile. From Mortimer's point of view, the notion of conceptual profile allows us to consider the teaching and learning processes from a new perspective. Learning requires students to overcome some limitations in order to change their profile —which is what Mortimer means by "conceptual change": The conceptual profile change. Teaching, on the other hand, requires the conceptual profile to identify those epistemological and ontological obstacles students face and act accordingly.

To construct the conceptual profile for such an abstract chemical concept, all these considerations were taken into account as well as the necessity to show several zones of it, each one having categories with more explanatory power than its precedents. It was decided to widen the spectrum of possibilities into the following five conceptual profile zones:

i) **Perceptive/intuitive.** This zone includes ideas on «amount of substance» corresponding to immediate impressions, sensations and intuitions, lacking structure or systematization. The concept of the "chemist's dozen" belongs to this zone because of its simplicity. Ideas that result from subjective and personal reflection are included in this zone because they constitute simple everyday life experiences;

ii) **Empiricist.** The notions of «amount of substance» determined objectively and precisely by the use of empirical scales, such as the mass or the volume of a definite quantity of substance, are placed in this zone. «Amount of substance» may be better perceived by students by means of the measurement of either the molar mass or the molar volume, as both concepts are closer to their everyday perception, evidently macroscopic in nature;

iii) **Formalist.** This zone is characterized by the use of algorithms and mathematical formulas as tools of analysis, applied without a complete understanding of the conceptual relationships involved. The «mole», mainly used to perform stoichiometric calculations, is devoid of any clarity of what its corresponding magnitude represents;

iv) **Rationalist.** It comprises ideas about «amount of substance» that imply a closer look of the atomic-molecular level. The discourse is fundamentally built around the nanoscopic vision of «amount of substance» expressed in terms of «number of elementary entities», without taking into account the macroscopic understanding of the concept; and

v) **Formal rationalist.** In this zone, "amount of substance" consists of a conceptual network, not merely the result of primitive, immediate and empirical experience. In this network, aspects of a macroscopic measurement of mass or volume are linked with the counting of a certain number of intangible entities. This zone sustains a coherent and balanced relationship between the macro and the nanoscopic levels of explanation. The finding of a formal rationalist zone implies that a scientific model of «amount of substance» has been acquired.

Results

CoRes of Interviewed Professors

After defining a set of six central ideas, professors were asked to fill the CoRe matrix. It took them between three and four weeks to return their results. In this section the highlights of the CoRes are presented.

1) There is a notable proclivity to talk about the «chemist's dozen», that is, to relate a numerical quantity to the concept of «mole». Consider this passage from the book of Herron *et al.* (1987, P. 98): "A dozen is a convenient unit for expressing a frequently used quantity. However, one or two dozen atoms are too small to be seen with even the most powerful microscope. The term «mole» is used to talk about a given number of atoms, molecules, ions, or electrons, just as a dozen is used to talk about a number of eggs, oranges, or doughnuts."
We have selected the following two paragraphs from professor 1, where she advises the use of big amounts such as «a pile» or «a bunch» as parallel terms of «a mole», even suggesting the following analogy which relates the molar mass with that of a dozen objects:

> "In general, I suggest to students troubled with the concept of «mole» to try and substitute this word for «a pile» or «a bunch». (It would be much easier to illustrate the concept by substituting "mole" for the cruder «a hell of a lot of», but then I would be judged by my students as lacking education. However, students are free to use it at ease)."
>
> "One of the everyday objects Mexican students count by the dozen are «tortillas»[1]; the analogy of molar mass with the dozen mass seems to work well."

2) About the tendency to link «amount of substance» with a mass (Dierks, 1981), as stated in Ostwald's definition, professor 2 pointed out that one of the problems is:

> "Making students understand that a mole implies the measurement of elementary entities by the determination of their mass".

And in relation to the fifth question of Table 1, professor 2 identified as the main hurdle:

> "The belief that an amount of substance is a mass of substance."

We insist that it has been mentioned by Furió *et al.* (2000) that among teachers and within textbooks the «mole» concept is wrongly understood as a chemical mass and/or a number of elementary entities.

3) Professor 3 wisely introduced the equivalentist and the atomist paradigms of the «mole» concept —we will refer to them in a subsequent section, within "Implications for teaching"— in the third question of table 1, when she said:

> "I am aware of the transformations the concept of mole suffered when changing from an equivalentist to an atomist point of view... Furthermore, I understand that the concept of mole arose within an equivalentist framework, in Ostwald's lifetime, as 'the mass in grams numerically equal to the molar mass', and that the concept of amount of substance acquired during the atomist conceptual framework, during 1961, the definition accepted nowadays. This is a very rare case in which the unit (the mole) was introduced and defined years before its magnitude (amount of substance)."

4) Professor 1 correctly insists on the separation of «amount of substance» from the concept of elementary entities. When she teaches «amount of substance» she does not even mention Avogadro's constant. In her answer to question one of table 1, she wrote:

> "In my opinion, Avogadro´s number is an overrated chemical constant. Commonly, its numerical value is overemphasized in detriment of the mole concept. I illustrate chemical formulas through an explanation of the concept of mole, and to do this it is unnecessary to introduce Avogadro's constant".

[1] Thin Mexican pancake made with corn flour, usually eaten hot and filled with almost anything as a substitute of bread.

In sharp contrast with this point of view, we selected the following phrase provided by professor 3, where she shows she understands that «amount of substance» is related to the «number of elementary entities», but seems unable to discern the difference between both terms, employing one for the macroscopic level and the other for the nanoscale one:

> "The teaching procedures that I use to engage pupils with this idea are based on calculations that allow them to understand the usefulness of counting very small particles such as atoms and molecules with the magnitude «amount of substance»".

Professor 4 also confuses «amount of substance» with «number of elementary entities», when she writes:

> "I consider essential for students not to confuse amount of substance with mass; for this purpose I include assessment problems in which they are required to calculate the number of elementary entities in equal masses of different substances and the mass of different substances that contain the same number of elementary entities."

In spite of the confusion, applying the assignment as suggested seems sound teaching practice with the aim of establishing a clear distinction between «mass» and «number of elementary entities».

5) Unfortunately, it is commonplace for professors to confuse the terms «Avogadro's hypothesis» — the relationship between volumes of gas, at the same temperature and pressure, and their «amount of substance»— with «Avogadro´s constant» — the inverse of which is the constant of proportionality between number of elementary entities and «amount of substance». We have an example of this misunderstanding in professor 1's response:

> "There is no trouble in establishing the relative mass of objects that can be seen and counted; however, the extrapolation of this concept to intangible objects, such as atoms, requires the understanding of Avogadro's hypothesis and a subtler level of abstraction by pupils."

And another example of the same kind, with historical confusion added, in professor 2's response:

> "Avogadro´s hypothesis consists of what is now known as Avogadro´s constant. The value of this constant can be calculated by diverse methods."

6) Another common confusion arises between «molar mass» and «molecular mass». For example, when professor 2 writes:

> "Once the mole concept is understood it is easier to grasp molar mass. However, sometimes confusion remains when the molar mass coincides with the molecular mass."

She seems to have forgotten that the numerical values may be the same in both magnitudes, but their units differ.

7) To illustrate that the participant professors rarely use the term «amount of substance», and that they frequently confuse it with «amount of matter» —a common mistake made by both secondary and university level teachers— we selected the following passage belonging to professor 1:

> "Although basically Avogadro's hypothesis is a gesture of good faith, due to the fact that it is based on the law of volumes' combination and the ideal gas equation, it is possible to experimentally illustrate it by evaporating different quantities of water in a closed environment of variable volume (such as a balloon)

> and showing that the volume of water vapor is proportional to the «amount of matter»".

Another mistake can be culled from this paragraph, wherein the professor confuses «Avogadro's hypothesis» with «Avogadro's law» —of proportionality between volume and «amount of substance» in a gas sample.

Classification of Sentences of the CoRes in Conceptual Profile Zones

To build the conceptual profiles, the hermeneutic process was used as research method. Once the four professors had written their respective CoRes, they were read with care to analyze their contents. A first reading provided a general idea of what should be expected from each of them. Afterward, each cell of the individual CoRes was tested so as to determine their place in any one conceptual profile zone (sometimes one matrix square did not include a classifiable sentence or, on the contrary, include more than one idea; in the latter case the zone that fitted the idea best was searched for, or alternatively, the cell text was divided into two or more phrases and then classified in different zones). This process was developed by at least two of the authors of this paper until an agreement was reached.

Below follows a selection of some of the sentences classified by profile zone so as to exemplify the construction of the conceptual profile graphs.

1) Some of the surveyed professors emphasized the quantitative aspect of «amount of substance». Several relatively old papers insist on this point [the initial sentence of Kolb's (1978, P. 728) paper reads: "one of the main reasons the mole concept is so essential in the study of chemistry is *stoichiometry*"], a concern compatible with the formalist zone of the conceptual profile. Professor 1 insisted on the mathematical relevance of the concept of «mole» when answering the following question included in the introduction of the CoRe frame "What is the importance given by you to the topic "amount of substance" and its unit the «mole» as part of the General Chemistry course?":

> "It is indispensable for pupils to understand precisely the concept of mole because all stoichiometric relationships are based upon it".

This phrase clearly emphasizes the formalist zone of the conceptual profile.

2) Professor 3, in relation to question one of Table 1 declared the following about the central idea of "mole, the unit of amount of substance":

> "It is important for students to know that the mole is the unit of one of the seven fundamental magnitudes of the SI: that of amount of substance. I try to make them understand that the mole is used in chemistry to carry out mathematical operations that allow chemists to determine the composition of substances expressed as a percentage, in the form of its formula, and what kind of relationship is there between the masses of reagents and products in a chemical reaction".

This last paragraph distinctly belongs to the formalist zone of the conceptual profile, answering as it is a question of the CoRe under the heading of objectives of teaching. As can be seen, in some cases the use of either «amount of substance» or «mole» is deemed necessary exclusively for stoichiometric calculations, without concern for arriving at any qualitative comprehension of the concept.

3) Another approach on this topic was supplied by professor 1, regarding the same question and the same central idea. She emphasized the «mole» as a unit devised to count atoms, not to perform calculations:

> "The mole is a unit used on a daily basis in chemistry; instead of counting atoms one by one, we chemists count them by moles. Thus, the correct manipulation of this concept is of fundamental importance in the professional training of chemists. I try for students to understand the mole in the same way they understand pairs, tens and hundreds, words which identify precise and finite number of objects. The number [of Avogadro] is not essential; it is enough to learn that the number of hydrogen atoms in approximately 1 g of hydrogen gas is the same as the number of chlorine atoms in 35.5 g of chlorine gas".

This statement has been classified in the rationalist zone of the conceptual profile, because it draws attention to the nanoscopic view of chemical systems, although the "pairs, tens and hundreds" part of it was agreed should belong to the perceptive/intuitive zone. This professor has revealed to us the way in which she teaches the main part of the topic «amount of substance»: by determining the relative masses of common objects and then using arguments such as those espoused by Arce de Sanabia (1993). We have included a PaP-eR of this type in the Appendix 2 of this paper.

4) Professor 2 stressed the mathematical relevance of the concept of «mole»:

> "It is the central topic of quantitative chemistry; it takes part in the majority of chemical calculations. It is a fundamental unit of measurement of the SI. Although in many current textbooks the concept of «chemical equivalent» is excluded, I consider it to be linked to the concept of mole because historically I was unable to understand how Mendeleiev had made use of the relative atomic masses as arranged by Cannizzaro without starting from equivalent weights".

This phrase also emphasizes the formalist zone, but it does more than that, because it reveals the long gone controversy between the atomistic and the equivalentist visions of chemistry (Padilla and Furió, submitted for publication).

5) Professor 4's answers have been classified mainly in the formal rationalist zone, as will be seen shortly. Let us take a look at her answer to the first question of the CoRe concerning the central idea of 'amount of substance as fundamental magnitude of the SI':

> "It is essential that students manipulate the units of the SI. Particularly, the unit for amount of substance is fundamental for experimental activities. I attempt for them to learn that the mole allows the counting of elementary entities in an indirect way, from macroscopic measurements".

This passage was classified as formal rationalist because of its consistent and unbiased relationship between the macro and nanoscopic levels of explanation. Novick and Menis (1976) have already remarked upon the long sought desire for students to elucidate the interactions between macroscopic measurements and microscopic interpretations. In this study we have included a PaP-eR of Professor 4 by gotten together a set of analogies used by her in everyday classroom practice in Appendix 1 of this paper.

6) To illustrate the way in which the answers of the CoRe were assigned to other conceptual profile zones, we took the 'molar mass' central idea and question 3 of the CoRe (Table 1), where professor 4 indicated the following:

> "In 1900, Ostwald defined the mole of a substance as its molar mass expressed in grams and numerically equal to its relative molecular mass. Epistemologically speaking, Ostwald would have to be considered an equivalentist".

This text was classified as empiricist, because it deals with the origin of the empirical scale with which to objectively determine the magnitude of the «mole».

7) As a further example of the intuitive/perceptive conceptual profile zone, the following statement contains immediate impressions, sensations and intuitions, without structure or systematization. It was provided by professor 3 in the teaching procedure question of the CoRe (number 7 in Table 1), on the central idea of 'relative atomic mass':

> "The teaching procedures that engage students with the topic being taught are, for example, the use of analogies accompanied with measurements of commonplace magnitudes such as mass of fruits, coins, seeds or paper sheets: Things they often come into contact with or consume. I make them determine the relative masses of those things".

As it has been previously pointed out by Ainley (1991), problems exist among students with the use and meaning of the word «relative», maybe because it is the first time they have seen such a word attached to the concept of mass. To remedy this situation, the use of analogies with commonplace objects, as suggested by professor 3, seems an appropriate course of action.

Once every sentence was located to a different profile zone, a countdown of the number of sentences belonging to each was performed. The results are presented in Table 2.

Table 2. Number of sentences each professor located in each one of the conceptual profile zones.

Professor	Perceptive/ intuitive	Empiricist	Formalist	Rationalist	Formal rationalist	TOTAL
1	12	16	4	5	2	39
2	5	15	9	10	4	43
3	9	15	8	6	9	47
4	3	9	4	8	16	40

Two PaP-eRs

The professional and pedagogical experience repertoires (PaP-eRs) constitute the second tool advocated by Loughran *et al* (2004) to document PCK, and consist of narrative explanations of a given piece of subject matter content from professors' actual practice. PaP-eRs offer a way to apprehend PCK's holistic nature and complexity. They have the ability to express a "discursive whole" by explaining through a text what a teacher considers as fundamental actions when teaching. A PaP-eR must allow us to take a look inside a teaching/learning situation where the content shapes the pedagogy. Therefore, the PaP-eR tends to be linked to only one or two cells of the CoRe, reflecting the richness of teacher's PCK.

A couple of PaP-eRs have been included in the appendixes of this paper, both of them about analogies used in actual teaching scenarios. The PaP-eR developed by professor 1 —inserted as appendix 2— deals with the concept of relative mass applied to common objects (such as nuts and bolts) from which an interesting «mole» simile can be derived. The other PaP-eR contains several analogies used by professor 4 in the teaching of her classes —inserted as appendix 1.

Gabel and Sherwood (1984) have argued about the importance of analogies, presenting themselves a set of them thought out with smaller objects, so as to overcome the visualization problem presented by large numbers of entities. Staver and Lumpe (1993) have for their part discussed the convenience of working with abduction, or reasoning

by analogy, in an effort to establish a connection between 6.02×10^{23} and a less cumbersome and more familiar counting unit, such as the dozen.

Implications for teaching

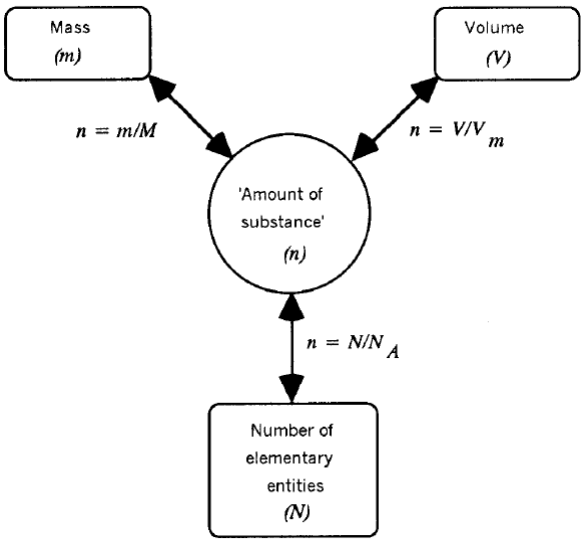
Wrong Use of the «number of moles».

In the practice of teaching, the «number of moles» is used either instead of «amount of substance» or associated with the generic expression «amount of matter», the latter mainly accepted as a mass. "The physical quantity «amount of substance» should no longer be called «number of moles», just as the physical quantity «mass» should not be called «number of kilograms»." (Mills *et al*., 1993). The term «amount of substance» is seldom used in textbooks (Staver and Lumpe, 1993; Furio *et al.*, 2000), and the consequences of this omission are reflected in the sporadic use of the term in the CoRes questionnaire answered by the interviewed professors, although all of them used the concept of «mole». The proper term, «amount of substance», should be used instead of «number of moles». None of the four professors made use of the term «number of moles» in their respective CoRes.

Suitable Use of all the Variables Involved.

A second, deeply relevant, implication of the present study for teaching is the recommendation of properly handling of the different variables that constitute each topic. In Figure 1 (taken from Furió *et al.*, 2000, but first included by Kolb, 1978), the corresponding operative expressions that relate «amount of substance», *n*, with «mass», *m*, «volume», *V*, and «number of elementary entities», *N*, (in the figure, *M* represents the molar mass, V_m the molar volume and N_A Avogadro´s constant) are indicated (for a more complete description of all variables see Strömdahl *et al.*, 1994, P.23).

Figure 1. Relationship among the variables «amount of substance», *n*, «mass», *m*, «volume», *V*, and «number of elementary entities», *N*.



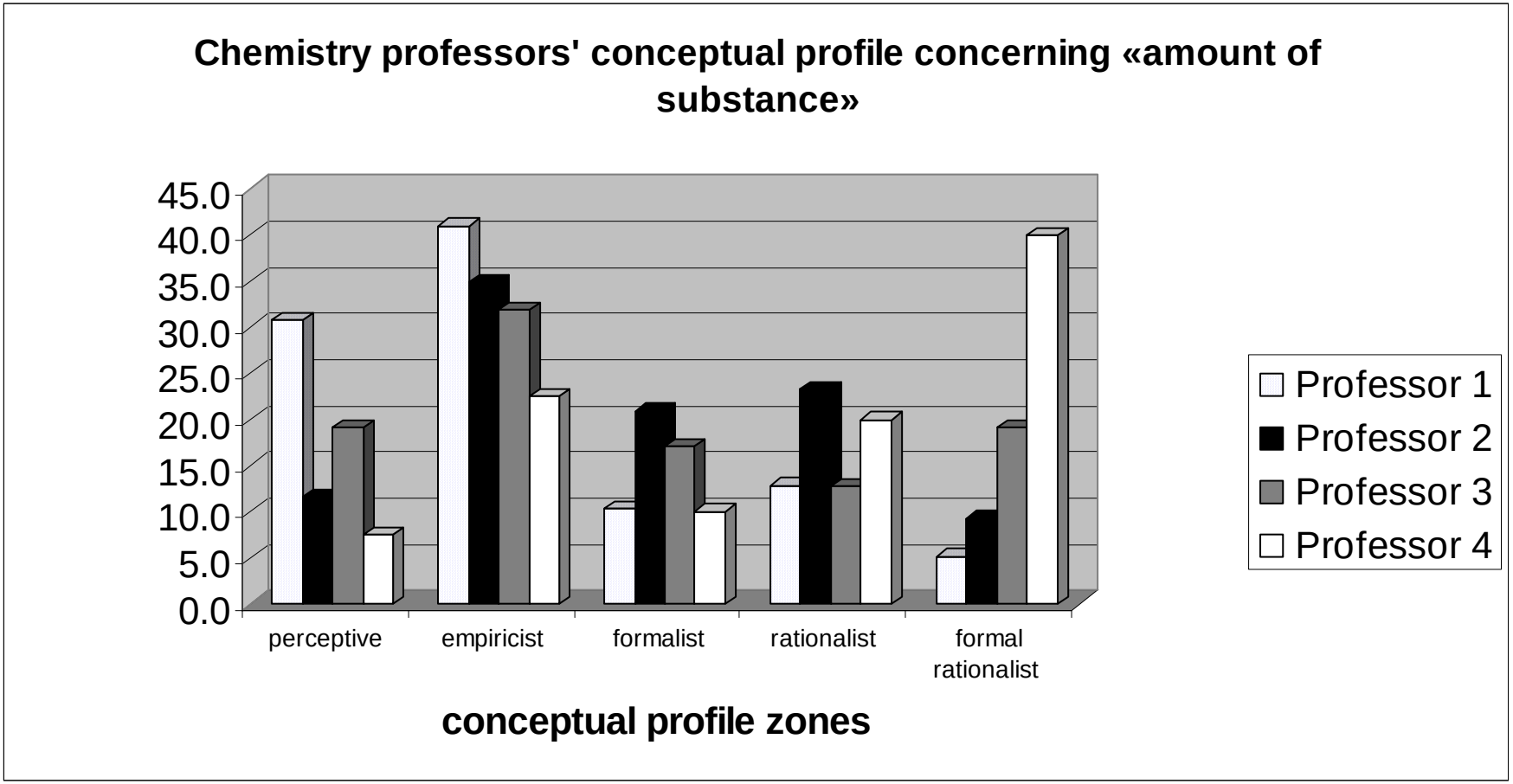
Equivalentist vs. Atomist Paradigms.

At this point of the discussion, the analysis can be further restricted since, historically, the concept of «amount of substance» had been seen through the prism of two paradigmatic conceptual frameworks: The ***equivalentist*** paradigm that chose mass as a way of representing "amount of substance", thus denying or ignoring the existence of atoms and molecules, and the ***atomist*** paradigm that considered as real the possibility of counting atoms and molecules and associated a macroscopic representation to it (Padilla and Furió, submitted). From this historical point, and relative to the five zones referred above, it is possible to say that the concept of «amount of substance» is basically governed by two ways of thought: The empiricist and the formal rationalist conceptions — reducible to the equivalentist and atomist views, respectively— fundamental to the process of thinking and teaching this concept.

It might be supposed that both paradigms are incommensurable. The idea of incommensurability, according to the etymology of the word and its use in mathematics, suggests that a pair of incommensurable things do not share the same standard of measure. Nevertheless, Webster's Dictionary (1971, p. 1143) includes a second meaning of the word "incommensurable" that extends well beyond the concept of measure: "Lacking a common basis of comparison in respect to a quality (as value, size, excellence, etc.) normally subject to comparison".

Kuhn (1970) introduced the idea of incommensurability when he proposed that, during a scientific revolution, an old paradigm is replaced — in whole or in part— with an incompatible new one. Incommensurability then is a property that involves deep semantic and conceptual obstruction between competing scientific theories. Two rival paradigms can be thought of as incommensurable if they assign different meanings to key concepts and possess different methods and standards, as is the case with the emphasis on mass in the equivalentist paradigm and the emphasis on particles in the atomist one.

Figure 2. The four conceptual profile graphs, expressed as the percentage of answers from each professor on the conceptual profile zones; from the mainly empiricist (or equivalentist; professor 1) to the mainly formal rationalist (or atomist; professor 4).



Conceptual Profile Graphs

It can be argued that the percentage of times a professor response was allocated into a specific conceptual profile zone represented a path of classroom thought. The present study found that there are at least two opposing ways to teach the topic "amount of substance". These range from the equivalentist paradigm — represented by professor 1 in Figure 2, armed with a radical and empiricist way of thinking, sustaining all her arguments on the fact that the relative atomic masses provide the same number of particles for any two samples of different substances, no matter what that number may be— to the atomistic paradigm —a formal rationalist zone represented by professor 4, for whom «amount of substance» is a conceptual network that involves macroscopic measurements of mass or volume, and simultaneously the counting of a finite and defined number of elementary entities.

Professor 1 seeks to teach «amount of substance» through concepts such as mass, volume and others associated with perception: A concept of «amount of substance» based fundamentally on counting entities belonging to the nanoscopic world from measurements —of mass and volume— made in the macroscopic world. However, she explicitly denies or refuses to mention a nanoscopic level of the concept —as was the case with Ostwald's definition of «mole». Professor 4 on the other hand delivers a class in accordance with the atomist paradigm. She almost always appeals to the right concept, although sometimes have a slight tendency towards empiricism, but avoiding any mention of «amount of substance»'s relationship with stoichiometric calculations (the formal zone).

In the middle of the classification lie two of the professors (numbers 2 and 3) that suffer from a tendency towards empiricism, with shades of the formalist and rational formalist zones. More specifically, Professor 2 shows a clear tendency towards empiricism, while her teaching strategies seem inclined in the direction of the formalist and rationalist zones, because she sometimes made reference to the nanoscopic way of thinking when talking about the concept of «mole». Professor 3 possesses the most heterogeneous way of thinking. She scored almost the same number of times in all five conceptual profile zones. Nevertheless, she scored a slightly higher percentage on the empiricist and perceptive zones than the other three professors, which reveals a tendency towards empiricism.

As a result of the present study, a new way of classifying the five conceptions of one «mole», reported by Swedish educators (Strömdahl *et al.*, 1994), has been developed, as seen in Table 3. Category F_4 in the table was not found by those authors in any of 28 educators that participated in their study, but was nevertheless included to complete the set of categories. Nelson (1991) has done work on the «standard molar mass», $\mu = 1 g\ mol^{-1}$, a mathematical quantity that allows the calculation of the «amount of substance» without needing Avogadro's constant. We dare suggest a reformulation of this last category, stating that "One mole is only important for performing stoichiometric calculations".

Table 3. Connection between the conceptual profile zones of the present study and Strömdahl *et al.*'s conceptions concerning one mole.

Category	Strömdahl *et al.*'s description	This work classification
F_0	One mole is a portion of a substance	Perceptive/intuitive
F_1	One mole is an elementary entity of specific mass	Empiricist

F_2	One mole is equivalent to Avogadro's number	Rationalist
F_3	One mole is a unit of the physical quantity amount of substance	Formal rationalist
F_4	One mole is excluded and replaced by a number (identical with Avogadro's) as the conversion factor between the two units of masses 1 μ and 1 g.	Formalist

It needs to be pointed out that the conceptions F_0, F_1, and F_2 were identified by Dierks (1981) in papers published prior to 1957.

Conclusions

Loughran *et al.*'s methodology to capture PCK seems to be a complete and interesting way of documenting and portraying this construct. We can only argue against this methodology the fact that it does not reflect the «eros» in the educational stage; some questions or comments have to be included, at least in the PaP-eRs, related to the affective portion of lecturing, due to its extreme importance.
On the other hand, Mortimer's conceptual profile seems a useful tool for sorting the epistemological and ontological affirmations of individual professors. It can be used to efficiently characterize the PCK of different professors and represent their commitments to the discussion of divergent ways of teaching the studied concepts. The sorting in conceptual profile zones of the statements written by professors in their CoRes, represents a new tool that proved to be very useful, especially in the classification of professors' ways of thinking and behaving in the classroom.
Furthermore, a second categorization, closer to chemistry's history of isolating professor's thoughts into two incommensurable paradigmatic visions of the teaching of « amount of substance» that stood in direct opposition during the whole of the nineteenth century, have been proposed: the equivalentist, based on the principle of selecting masses (named equivalents) as a means of representing quantitatively chemical reactions, but denying or ignoring the existence of atoms and molecules — corresponding to the empiricist zone of the conceptual profile— and the atomist, based on the belief that atoms and molecules do exist and are useful entities to clarify the occurrence of chemical reactions, taking for granted the existence of this kinds of particles and the possibility of counting them indirectly through a real macroscopic representation —corresponding to the formal rationalist conceptual profile zone. This dichotomy has also been found on the interviewed professors.
The following conclusions stem from the analysis of the CoRes of the interviewed professors and Mortimer's conceptual profile analysis:

- Educator's lack of knowledge of the historical context of the concepts tested is partially to blame for the difficulties in teaching them.
- It is easier for professors — and, of course, for students— to visualize «mass» or «volume » instead of «amount of substance», because those two quantities are closer to their everyday life and intuition. This fact explains teacher's tendency to provide statements that fit squarely inside the empiricist zone of the conceptual profile.
- «Amount of substance» is a magnitude that has been incorrectly understood, and is seldom used by most professors. Neither is it included in most General

Chemistry textbooks. It seems pertinent to recommend the inclusion, in textbooks, of the definition given by IUPAC (McNaught and Wilkinson, 1997; pasted at the "Content" subsection of the "Methodology" section of this paper). The generalized use of «chemical amount» instead of «amount of substance» is essential because on occasions the amount specified does not bear any relation to a specific substance, but it is always related to chemical entities —as ions, electrons, substituent groups, ligands or radicals.

- Each professor's experience, made up of beliefs and biases, fixes the bounds within which she feels comfortable, but at the same time favors and enhances her teaching practice.
- The five conceptual profile zones analyzed in the present study represent a new way of classifying the five commonly expressed conceptions of educators, as reported by Strömdahl *et al.* (1994).

Appendix 1: PaP-eR on the Use of Analogies to Teach «amount of substance» and Other Chemistry Concepts

Professor 4 holds the chair of a Chemistry I course at a Diploma degree program. The weekly, evening shift course is 5 hours long and operates with a semivirtual modality. Students attend class sporadically (an average of one hour a week). Every week the professor proposes a new subject via the virtual Campus, with accompanying explanations, examples and exercises to be solved by students, as well as a suggested bibliography. The virtual Campus has a comments forum called “Debates” where students and teachers alike share ideas, discuss issues with their classmates and build their own analogies and visualizations, with supervision in order to insure a better understanding of the subject.

The presentation of topics in attractive and friendly ways, as well as the provision of analogies, examples and applications, has proved to be a key aspect of this kind of teaching. Here, we analyze how professor 4 delivers her lecture on «amount of substance» and how she weaves a set of analogies in the fabric of it.

When talking about the importance of taking measurements and relaying those figures in such a way that everybody can understand them, professor 4 pointed out:

> “You all know how to count the coins in your purses or pockets, or the goals in a football match. You can measure your weight using a scale, your height with a ruler, the time with a clock, the volume with a measuring cup, the pressure with a manometer (like those used with regard to tires at gas stations) and the speed of an automobile with a speedometer. You also know that every measure has two components: a magnitude and its measuring unit. You do not say: ‘from home to University there are three’, because the person listening to you will surely ask: ‘three what?’

Then professor 4 adds:

> “In order for people to reproduce a measurement of any magnitude, it is necessary to establish standard units. In daily life, people use the metric system: centimeters, meters, kilograms, tons, etc. Worldwide scientists use the SI units (International System of Units).

The professor then goes on to describe the system. Later in the day, in order to better explain the concept of isotope and how it affects the calculation of atomic weights, professor 4 explains the reason behind the need to calculate the weight average of isotopic masses taking into account the percentage of natural occurrence of each isotope by means of an analogy with the calculation a student’s final average grade from their partial marks for the course.

Later in class, when professor 4 wishes to show the importance of working with relative masses, she wrote:

> “The mass of a fish’s flake is too small to be measured by means of a top loading scale (capable of detecting a mass difference of 0.1g). But it is really possible to measure the mass of groups of fish flakes. If 100 large fish flakes weigh 4.74 g and 100 medium size fish flakes weigh 3.32 g, we can conclude that 100 large fish flakes are 1.43 times heavier than 100 medium fish flakes”.

$$\text{Relative mass of a large flake} = \frac{4.74\,\text{g}}{3.32\,\text{g}} = 1.43$$

Since the groups of fish flakes have the same amount of flakes, the mass of a large fish flake must be 1.43 times bigger than that of a small one. If the mass of a small fish is defined as a unit, then the mass of a large fish flake will be equivalent to 1.43 units".

The latter explanation is used as an introduction for the following concept:

> "The relative masses of different elements can be measured in a similar way: by weighing the mass of a fixed number of atoms. Since the relative mass of an atom is so small, the amount of atoms in the groups should be extremely large".

Professor 4 introduced then another interesting analogy concerned with the concept of relative mass of fruits in this instance:

> "If we weigh a certain amount of grapes and afterward weigh the same amount of plums, we will find that the plums are several times heavier than the grapes. Imagine that the relationship between the two masses is as follows:
>
> m plums/ m grapes = 8.0
>
> "That is, one plum is eight times heavier than one grape. The key question here is: 'If a certain amount of grapes weighs 100 g and you wish to have the same amount of plums in a salad, how do you solve this problem just by weighing?' Students notice right away that 800 g of plums will have the same amount of individual plums as 100 g of grapes. Briefly, we conclude that the relative masses help us to 'count' by weighing.
>
> "Similarly, the amount of substance in the SI is measured in moles. A mole of any substance is a sample of the substance that has as many elementary entities as atoms we can find in 0.012 kg of Carbon-12. Then, by definition, one mole of atoms of carbon–12 has a mass of 12 g, and thus one mole of atoms of any other element will have a mass in grams numerically equal to the relative mass of that atom in a relation of 1/12 to the mass of the atom of carbon-12. Let us consider the underlined phrase. The basis of this scale is 1/12 of the mass of one carbon-12 atom. If, for example, we are dealing with a sample of silver atoms, with a relative mass of 107.8682 in this scale (each silver atom weighs approximately nine times more than one carbon-12 atom. Since $9 \times 12 = 108$, this is the relative mass of the silver atom). As a result, in 108 g Ag there are the same numbers of atoms present in 12 g of carbon-12, each one weighing nine times that of 12C".

The analogy of grapes and plums explains why the relative mass of any atom in the periodic table (expressed in grams) can lead us to quantities of substance that have the same number of elementary entities as 12 g of ^{12}C.

Professor 4 then goes over the Avogadro's constant. She tells her students the following:

> "Imagine that you sell vegetables in large quantities, and a grocery store needs 20,000 green peas. Can you imagine how much time it would take you to count, pea by pea, until you reach 20,000? However, if you know, for example, that 1,000 green peas weigh approximately 100 g, you will only have to weigh 2 kg of peas and send the order to your customer".

Later on she adds:

> "It is impossible to count atoms individually, because atoms cannot be seen at plain sight. In this case we need a unit that connects the submicroscopic world of atoms and molecules with the macroscopic world. In the same way that 'pairs' and 'dozens' are appropriate units of measurement for socks and eggs, respectively, the mole, the SI unit for amount of substance, represents a quantity of atoms, molecules or ions. Using the mole as a unit of amount of substance we

> are able to count submicroscopic particles such as atoms, molecules and ions, by measuring the mass of them all."

As the class session progressed, professor 4 wrote:

> "It has been experimentally determined that the quantity of elementary entities in one mole is equal to 6.022137×10^{23} particles. This quantity is known as Avogadro's constant."

To provide students with an idea of the dimension of this number, she points out:

> "Avogadro number is very big, huge. If you place coins of $1 side by side (in our country coins measure 2.2 cm of diameter) until you reach Avogadro's number, the pile of coins would go around the Equator (the equatorial perimeter is 40,077 km) 330 billon times (3.3×10^{14}).

She also gives them the following information:

> "On the other hand, if you wished to measure one mole of iron atoms by counting them at the speed of one atom per second, it would take you 1.9×10^{16} years, a whole lot more than any human being's lifetime, since nobody has ever lived for 1.9×10^{14} centuries!"

And finally, professor 4 shows students some surprising calculations:

> "All of you know that water is essential for life. Water is one of the essential resources of Nature… however; maybe you are unaware that the total volume of water on Earth is 1,360 million km^3, that is to say 1,360,000,000,000,000,000,000 liters. 1.36×10^{21} liters is equivalent to 1.36×10^{24} ml. If we consider that one drop of water has an average volume of 0.05 ml, then the total amount of drops of water in the world is 27.2×10^{24} drops. That represents only 45.1 moles of drops!"

Appendix 2: PaP-eR on the Use of Scales to Register Relative Masses of Common Objects to Connect with the «mole» Concept

Professor 1 teaches General Chemistry, with a syllabus that includes the concept of «amount of substance» and its corresponding unit. However, this professor in particular prefers the use of a very interesting analogy related with the relative masses of bolts, nuts, washers and nails of different sizes. Professor 1 explains that he likes this analogy a lot because it explains the relationship between the relative masses of two different objects and the weights of the same number of objects of both kinds, without ever resorting to Avogadro's number. She believes this concept only causes confusion among students: It is way too large to make it understandable.

In what follows we will strive to describe this analogy as best we can. Professor 1 begins by mentioning relevant aspects of the History of Chemistry. These topics are then linked with concepts such as element and substance, which prove useful to arrive to the topic of molecular composition and its relationship with atomic mass and Avogadro's hypotheses.

To make use of the experimental analogy it is necessary to have at hand a considerable quantity of manufactured objects such as bolts, nuts, washers and nails (these sorts of objects have few flaws of manufacturing, so their masses are almost always the same for each kind, so students can build ensembles (in the manner of molecules) with them. What is also needed is a scales with which to measure equal masses of different objects without minding the exact magnitude of the measurement.

Over each dish of the scales professor 1 placed an unspecified amount of bolts: On one of the dishes the smallest bolts (we will call them "standard" type), and on the other dish the biggest ones in enough quantity to balance the scales. The purpose was to choose a standard reference, in this case the smallest bolt. Once the dishes were balanced, professor 1 began to count the number of bolts on each dish. With the smallest bolt as reference, she builds Table 4 (the number of individual bolts indicates their size, the higher the number the smallest the bolt).

Table 4. Results from which professor 1 obtains the following equation: $82m_{sta} = 30m_{(3)}$ or $647m_{sta} = 237m_{(3)}$ which implies $m_{(3)} = 2.73m_{sta}$.

Standard bolt	Bolt number 3
82 pieces	30 pieces
647 pieces	237 pieces

The purpose of counting every bolt is to demonstrate that no matter the mass of each type of bolt, the ratio of amount of pieces of one type of bolt to the other remains constant (2.73 in this case). If professor 1 wished to determine the mass of just one "number 3" bolt in terms of the mass of one "standard" bolt, she would have used the relationship $m_{(3)} = 2.73m_{sta}$.

With the above formula, the mass of one "number 3" bolt is equivalent to 2.73 times the mass of the "standard" bolt. The next step in her demonstration was to propose a new scale of measurement that adopted the "standard" bolt as reference, calling it the "Bolty". After repeating the same procedure for the other different types of bolts, professor 1 obtained the relationships shown in Table 5.

Table 5. If the "standard" bolt is the "Bolty", it is possible to calculate the relative masses compiled in the second column of the table.

Bolt type	Mass equivalent to the following number of Boltys (b = Bolty)
Sta	1 b
3	2.73 b
2	4.94 b
1	10.22 b

Now professor 1 wants to construct a base for counting bolts. She selects 1 gram of "standard" bolts (or Boltys) as the counting reference. ¿What are the weights of type 1, 2 and 3 bolts that contain the same number of bolts as the reference? The answer is given in Table 6.

Table 6. The relative masses of Table 5 can be used to weigh amounts of bolts types 1, 2 and 3 that have the same number of bolts as 1 gram of Boltys.

Bolt type	Mass of bolts with the same number of Boltys
Sta	1 g
3	2.73 g
2	4.94 g
1	10.22 g

The problem remains that if she insists on selecting the counting reference as 1 gram of Boltys, a small weight that only contains 2 or 3 bolts, an error is introduced in the procedure. To address this issue she then choose 1 ounce of Boltys as the reference for further counting, transforming Table 6 into Table 7.

Table 7. Amount of bolts types 1, 2 and 3 that have the same amount of bolts as 1 ounce of Boltys. All samples have the same number of bolts because the ratio between two of the masses is the same as the relative mass of the objects.

Number of bolts	Boltys (b= 1Bolty)	Ounces (oz)	Grams (g)	Number of objects
Sta	1 b	1	28.35	79
3	2.73 b	2.73	77.40	79
2	4.94 b	4.94	140.05	79
1	10.22 b	10.22	289.74	79

She used two different scales (one based in a single Bolty and one based in one ounce of Boltys) to measure masses with the same quantity of bolts notwithstanding which scale was being used. Thus, her "mole" of objects would have the same quantity of objects independently of their masses. She called this mass the molar mass of bolts, a magnitude proportional to the relative mass of each bolt.
Something quite similar was developed with different metallic objects: Nails, nuts and washers (see Table 8). What she determined is a ratio between the masses of the pieces to the mass of the standard (in this case, a nail). Afterward, she asked her students what would happen if the mass of reference were increased, for example, from 1 ounce or

50.0 g to 1 ton. The conclusion reached was that the number of pieces would be the same if the relative mass is taken as a base to construct the samples as shown in Table 8.

Table 8. One nail is the standard in this hypothetical table. Samples of the masses shown in any of the three remaining columns have the same number of objects.

Type of metallic piece	Numeric ratio	"Naily"	Mass of reference		
			1 ton	50g	100 g
Nails (sta)	1/1	1 n	1 ton	50 g	100 g
Bolts	57/285	5 n	5 ton	250 g	500 g
Washers	100/400	4 n	4 ton	200 g	400 g
Nuts	20/200	10 n	10 ton	500 g	1000 g

Another important point to draw attention to is that professor 1 can then establish that in an ensemble of bolts, washers and nuts, the relative mass of that group of pieces would result from the sum of the relative masses of the individual ones. When she was convinced that all of her students had understood the fact that the number of pieces remains the same independently of the mass she was putting on the scales, she transferred the idea to the environment of chemistry using the hydrogen atom as a standard, just as it had been done historically. With hydrogen as a standard, then C would weigh the same as 12 atoms of H, Cl the same as 35.5 atoms of H and O the same as 16 atoms of H. The same relationships can be expressed in terms of mass by saying that the mass of one reference of hydrogen would weigh 1 g, the mass of carbon 12 g, that of chlorine 35.5 g and that of oxygen 16 g: All these masses contain the same number of atoms, no matter what that number may be. In this way, professor 1 did not have to mention Avogadro's constant; it is enough to know that the number of molecules, atoms or particles remains the same.

Next, she assembles pairs of nuts and bolts held together in order to simulate HCl molecules, and then explains that 36.5 g of the compound represent one mole. She holds one nut and one bolt with two washers inside, a molecule of $HClO_2$ with a molar mass of 68.5 g (1+35.5+2×16=68.5).

Finally, with the relative masses of atoms at hand, she determines the composition of elements, expressed as a percentage, in a given substance as the final problem.

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